Outline:-

Landau functional and Q.P. energy, Thermodynamics

QPs → do interact

→ but the way the interact is very classical i.e. density-density interaction. (constrained by pauli exclusion ofc)

$$F = E - TS$$

$$E = \sum_{p\sigma} \varepsilon^0_{p\sigma} n_{p\sigma} + \frac{1}{2} \sum_{\substack{p\sigma \\ p'\sigma'}} f_{\substack{pp' \\ \sigma\sigma'}} \, \delta n_{p\sigma} \, \delta n_{p'\sigma'}$$

$$\frac{\delta E}{\delta n_{p\sigma}} = \varepsilon^0_{p\sigma} + \sum_{p'\sigma'} f_{\substack{p\sigma \\ p'\sigma'}} \, \delta n_{p'\sigma'} \overset{\text{def}^n}{\equiv} \underbrace{\xi_{p\sigma}}$$

the effective QP energy

(i.e. it depends on distribution as well)

$$S = -\sum_{p\sigma} \left[n_{p\sigma} \log n_{p\sigma} + (1 - n_{p\sigma}) \log (1 - n_{p\sigma}) \right] \times k_B T$$

$$\frac{\delta S}{\delta n_{p\sigma}} = -\log \left[\frac{n_{p\sigma}}{1 - n_{p\sigma}} \right]$$

$$\therefore \frac{\delta F}{\delta n_{p\sigma}} = 0 \Rightarrow \boxed{n_{p\sigma} = \frac{1}{e^{\beta(\xi_{p\sigma} - \mu)} + 1}}$$

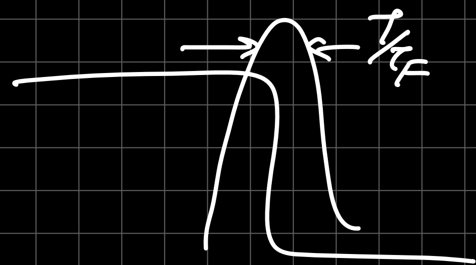
Integral eqn

How do we proceed? One strategy could be to linearize this eqn at low Temp.

Low temp expansion :-

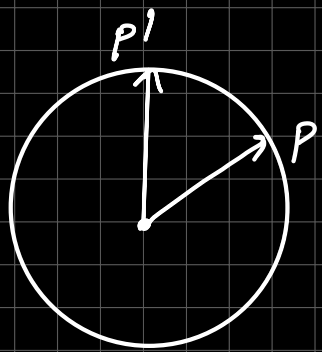
* not many excitations

$$n_{p\sigma}(T \to \text{very low}) = \frac{1}{e^{\beta(\varepsilon^0_{p\sigma}-\mu)}+1} + \left(\frac{\partial n_F}{\partial \mathcal{E}}\right)\Bigg|_{\mathcal{E}=\varepsilon^0_p-\mu} \sum_{p'\sigma'} f_{pp'\sigma\sigma'}\, \delta n_{p'\sigma'}$$



$T <<< \mu$

$$\frac{\partial n_F}{\partial \mathcal{E}} \sim \delta(\varepsilon^0 - \mu)$$



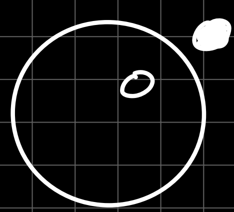
$\therefore$ in the $f_{pp'\sigma\sigma'}\, \delta n_{p'\sigma'}$, we can assume is a f^n of $\angle$ b/w p & p'

$$\sim \frac{\partial n_F}{\partial \mathcal{E}} \underbrace{\int p'^{\,d-1} dp'\, \delta n(p')}_{=0} \int d\Omega' f$$

$= 0$ from particle # conservation

from spherical symmetry

$$\int f(\theta-\theta')\ \delta n(\theta', p')$$



Thermodynamics

$$n_{p\sigma} = \frac{1}{e^{\beta(\varepsilon^0_{p\sigma}-\mu(T))}+1}$$

$$\frac{C(T,V,N)}{V} = \frac{\pi}{2}\frac{Tn}{E_F} \qquad \text{(see notes)}$$

$\hookrightarrow$ now E_F changes due to m^*

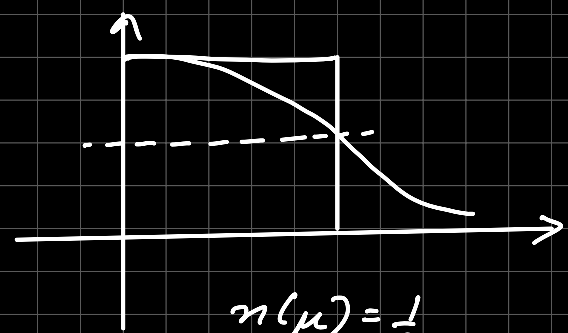
$E_F = \frac{P_F^2}{2m^*}$ $\quad$ P_F doesn't change $\Rightarrow$ Luttinger's theorem

$m^* \to$ generally $> m_e$ (difficult to move through a crowd)

$\therefore \frac{C}{V} \sim a T n^{1/3} m^*$

$\Rightarrow$ enhancement

μ (at large Temp reduces)



$n(\mu) = \frac{1}{2}$

$\epsilon_F - \frac{T^2}{E_F}$

$\mu \downarrow$ at higher temp. due to particle # conservation.

$$T \frac{\partial S}{\partial T} = \frac{C}{V} = a T n^{1/3} m^*$$

$$\Rightarrow S = a n^{1/3} T m^* + f(N, V)$$

from therm. 3rd law $= 0$

<u>maxwell's relation</u>

$$dF = -S dT + \mu dN$$

$$-\frac{dS}{dN} = \frac{\partial \mu}{\partial T} \Rightarrow \mu = \int dT \, \frac{1}{3} n^{-2/3} \times a T m^* \sim T^2$$

or $E_F - \frac{T^2}{E_F}$

$$\partial_T \mu = (-a) \left[\frac{1}{3} n^{-2/3} m^* T + n^{1/3} T \frac{\partial m^*}{\partial n} \right]$$

$$\mu(T, n) = \mu(T=0) - \frac{a T^2}{2} \left[\frac{m^* n^{-2/3}}{3} + n^{1/3} \frac{\partial m^*}{\partial n} \right]$$

extra term due to interaction

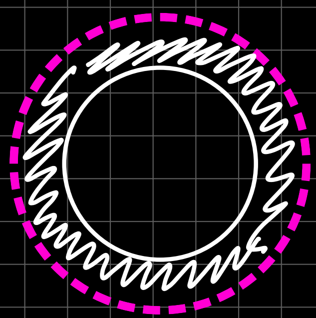
<u>point</u>:- Maxwell's relations are imp & convenient.

(Baym & Pethick has an alternative derivat^n)

Compressibility:-

* every measurement measures some kind of excitat^n.

Compressibility is a measure of expansibility of FS



$$f(\theta) \sim \sum_l f_l e^{il\theta} \quad d=2$$

$$\sum_l f_l P_l(\cos\theta) \quad d=3$$

$$f = f^s + f^a$$

susceptibility: $f^a_{l=0}$

* any measurement is akin to a perturbation. Pert. involves probe "f" or Land. param.

$$K = -\frac{1}{V}\frac{\partial V}{\partial P} = \frac{1}{n^2}\frac{\partial n}{\partial \mu}$$

$$n_{p\sigma} = \frac{1}{e^{\beta(\xi_{p\sigma}-\mu)}+1}$$

$\xi_{p\sigma} \to n_{p\sigma} \to$ depends on μ implicitly

$$\delta n_{p\sigma} = \frac{\partial n_{p\sigma}}{\partial \xi_{p\sigma}}\left[\delta\xi_{p\sigma} - \delta\mu\right]$$

$$\delta\xi_{p\sigma} = \sum_{p'\sigma'} f_{pp' \atop \sigma\sigma'}\, \delta(n_{p\sigma}\; n_{p'\sigma'})$$

$$\delta N = \underbrace{\sum_{p\sigma} \delta n_{p\sigma}}_{\text{adding particles}} = \sum_{p\sigma}\frac{\partial n_{p\sigma}}{\partial \xi_{p\sigma}}\sum_{p'\sigma'} f_{pp' \atop \sigma\sigma'}\,\delta n_{p'\sigma'} - \sum_{p\sigma}\frac{\partial n_{p\sigma}}{\partial \xi_{p\sigma}}\,\delta\mu$$

$$\frac{\partial n}{\partial \xi} = -\delta(\epsilon - \epsilon_F)$$

1st term:- $\displaystyle\sum_{\substack{pp' \\ \sigma\sigma' \\ E.F.S.}} f_{\substack{pp' \\ \sigma\sigma'}} \, \delta n_{p'\sigma'} = \underbrace{\int f(\cos\theta)\, d\cos\theta}_{\text{angular dependence}} \sum_{p'\sigma'} \delta n_{p'\sigma'}$

$d=3 \qquad f_{pp'} = \displaystyle\sum_{l=0}^{\infty} f_l \, P_l(\cos\theta) \qquad \theta = \angle \text{b/w } p \,\&\, p'$

$\int d\cos\theta \; P_l(\cos\theta) \neq 0$ only if $l \neq 0$

$= \underbrace{V \, D(E_F)\, f_{l=0}}_{F_{l=0}} \; \delta N$

$\therefore \; \delta N = \delta\mu \; D(E_F) - F^S_{l=0} (\delta N)$

$\delta N = \dfrac{(\delta\mu)\, D(E_F)}{1 + F^S_{l=0}} \qquad \therefore \; K = \dfrac{1}{n^2} \dfrac{\partial n}{\partial \mu}$

$= \dfrac{1}{n^2} \dfrac{D(E_F)/V}{[1 + \underset{\sim}{F^S_{l=0}}]}$

$D(E_F)$ → enhanced by $\frac{m^*}{m}$

$F^S_{l=0}$ ⇓ due to interactions

in expts, landau parameters can be determined expts

Spin-Susceptibility

$\xi_{p\sigma} = \dfrac{p^2}{2m^*} - h\sigma \, n_{p\sigma} + \dfrac{1}{2} \displaystyle\sum_{\substack{p\sigma \\ p'\sigma'}} f_{\substack{pp' \\ \sigma\sigma'}} \, \delta n_{p\sigma} \, \delta n_{p'\sigma'}$

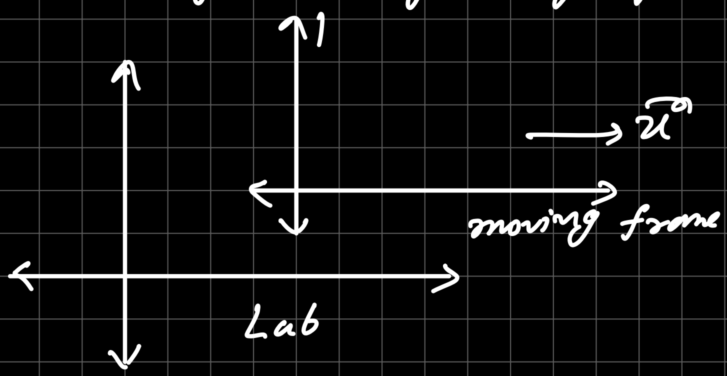
$\delta m = \displaystyle\sum_{p\sigma} \sigma \, \delta n_{p\sigma}$

$f_{\substack{pp' \\ \sigma\sigma'}} = f^S_{pp'} + f^a_{pp'} \, \sigma\sigma' \qquad \sigma\sigma' = \pm 1$ ↗ ∥ to zero, antiparallel

Check this result:- $\delta m = -D(E_F)\, v\, f^a_{\ell=0}\, \delta m + \delta\mu\, D(E_F)$

Effective mass m^*:

Nice argument by using galilean invariance.



$\vec{p}' = \vec{p} - M\vec{u}$

$\mathcal{H}' = \mathcal{H} - \vec{P}_{total} \cdot \vec{u} + \frac{1}{2} M_{total} \cdot u^2$

Ham in primed frame

momentum in lab

mass

derivat^n:-

$\frac{1}{2} m_i v_i^2 + \underset{\hookrightarrow \text{ indpt of velocity}}{\text{int}}$

$\rightarrow \frac{1}{2}\sum_i m_i (v_i - u)^2 + \text{int}$

Consider adding a bare particle with momentum $\vec{p}$

$\Delta E = \varepsilon_p$ (quasiparticle energy)

$\Delta \vec{p} = \vec{p}$ $\quad \Delta M = m$ ($\&$ not m^*)

$\Delta \vec{p}' = \Delta\vec{p} - (\Delta m)\vec{u} = \vec{p} - m\vec{u}$

$\Delta E' = \Delta E - \vec{u}\cdot\underset{\Delta\vec{p}}{\vec{p}} + \frac{1}{2}\underset{\Delta M}{m} u^2$

$= \varepsilon_p - \vec{p}\cdot\vec{u} + \frac{m}{2}u^2$ } new dispersion relation in primed frame

$$\xi'_{p-mu} = \xi_p - \vec{p}\cdot\vec{u} + \frac{1}{2}mu^2$$

$$\Rightarrow \boxed{\xi'_p = \xi_{p+mu} - \vec{p}\cdot\vec{u} - \frac{1}{2}mu^2}$$

How would you do it from a lab frame?